Apply Topological Study to A Meaning Based Information Theory

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Abstract

In previous works, we presented a meaning based information theory in which the core concept is an *informalogical space*. We did various discussions in informalogical spaces. We introduced *closed information intervals* and *open information intervals*, and based on which we introduced *closed neighborhoods* and *open neighborhoods* of a piece of information. Based on closed neighborhoods and open neighborhoods we introduced *closed convergence* and *open convergence* of *information nets*. We established Moore-Smith style convergence theory of information nets for closed convergence and open convergence. We also introduced *closed interval covers* and *open interval covers* of an informalogical space, and based on which we introduced *closed compactness* and *open compactness* of an informalogical space. We established relationships of *closed convergence* and *open convergence* with *closed compactness* and *open compactness*, respectively.

In this paper, we build two topological spaces in an informalogical space based on closed information intervals and open information intervals. The two topological spaces are called closed interval induced topological space and open interval induced topological space. We show that the closed convergence and open convergence of information nets in the informalogical space are the same as the Moore-Smith convergence of nets in the closed interval induced topological space and open interval induced topological space, respectively. We show that the closed compactness and open compactness of the informalogical space are the same as the compactness of the closed interval induced topological space and open interval induced topological space, respectively.

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With the closed interval induced topological space and open interval induced topological space introduced and the relationship between the original informalogical space and corresponding topological spaces established, we can apply more existing concepts and results in topological spaces directly to the original informalogical space. This can broaden and facilitate study of informalogical spaces and subsequent applications in real world problems. At the same time we also point the limitations of studies of the topological spaces comparing with studies of the original informalogical space from which the topological spaces are induced.

Key words: informalogical space, information interval, neighborhood, information net, convergence, compact.

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1. Introduction

In [1], we presented a meaning based information theory, a theory of information that is based on meanings of information and relationships among information. We introduced the core concept of our approach, *informalogical space*. We introduced the concept of an *information interval* in an informalogical space and, based on information intervals, we introduced the concept of a *neighborhood* of a piece of information in an informalogical space. We introduced the concept of an *information net* in an informalogical space. Based on the concept of neighborhood, we built a *convergence* theory for information nets in an informalogical space. The convergence theory is similar to the Moore-Smith convergence theory in general topology in that, for all major results of Moore-Smith convergence theory (see [4, Chapter 2]), we obtained similar results for the convergence of information nets in an informalogical space. In [2], based on the concept of information interval, we introduced the concept of *interval cover* of an information set in an informalogical space and, based on the concept of interval cover, we introduced the concept of a *compact* informalogical space. We proved that an informalogical space is compact if and only if each information net in the informalogical space has a subnet that converges.

In [3] we showed an undesirable property of closed convergence of information nets in an informalogical space $(\mathcal{S}, \mathcal{I})$: for an information net to closed converge to a piece of information I in the informalogy (*i.e.*, $I \in \mathcal{I}$), it is necessary that the information net be eventually identical to I. To avoid the undesirable property of closed convergence in an informalogical space, we introduced in [3] the concepts of *open information interval*, *open neighborhood*, and *open interval cover*, and based on those concepts we introduced *open convergence* of information nets and *open compactness* of informalogical spaces. After those concepts were introduced, we started to call the parallel concepts in [1] and [2] *closed information interval*, *closed neighborhood*, *closed interval cover*, *closed convergence*, and *closed compactness*.

Similar to closed convergence and closed compactness, we established in [3] a Moore-Smith style convergence theory for open convergence of information nets and proved that an informalogical space is open compact if and only if each information net in the informalogical space has a subnet that open converges. Open convergence avoids the undesirable property of closed convergence. However, we also pointed out the limitation of open convergence: for open convergence, a Moore-Smith style convergence theory cannot be established in general informalogical spaces, and the convergence theory can only be established in *open normal* informalogical spaces. An open normal informalogical space is an informalogical space in which the intersection of any two open intervals is still an open interval.

In this paper, we generalize the concepts of *closed information interval* and *open information interval* to obtain *generalized closed information interval* and *generalized open information interval*, respectively. We redefine various concepts in [1], [2] and [3] using generalized closed information intervals and generalized open information intervals. We prove that the redefined concepts are equivalent to the same named concepts in [1], [2] and [3] that were defined using closed information intervals and open information intervals.

With generalized closed information interval and generalized open information interval introduced, we would be able to build two topological spaces in an informalogical space based on generalized closed information intervals and generalized open information intervals. The two topological spaces are called closed interval induced topological space and open interval induced topological space. We show that closed convergence and open convergence of information nets in the informalogical space are the same as the Moore-Smith convergence of nets in the closed interval induced topological space and open interval induced topological space, respectively. We show that closed compactness and open compactness of the informalogical space are the same as compactness of the closed interval induced topological space and open interval induced topological space, respectively.

Furthermore, once closed interval induced topological space and open interval induced topological space are introduced, and relationship between informalogical spaces and corresponding topological spaces are established, we can apply more existing concepts and results in topological spaces directly to the original informalogical spaces. This will broaden and facilitate study of informalogical spaces and subsequent applications in real world problems. At the same time, we also point the limitations of studies of the topological spaces comparing with studies of the original informalogical space from which the topological spaces are induced.

Before introducing topological spaces in an informalogical space, we revisit some concepts that were introduced in [1], [2] and [3]. As the same as assumed in [1], [2] and [3], all the pieces of information under discussion are consistent information, meaning we cannot infer any contradictions from the pieces of information.

The *contain* relation between two pieces of information, and the *union* and *intersection* operations on information were introduced in [1]. Basically, suppose I and J are two pieces of information. If I can be inferred from J, then information I is *contained* in information J, and we can also say that information J *contains* information I. This relation is represented as $I \preceq J$ or $J \succeq I$.

As for the union and intersection operations on a non-empty information set $\mathcal{A}$, basically, the union $\vee\mathcal{A}$ is the sum of all the pieces of information in the information set $\mathcal{A}$, and the intersection $\wedge\mathcal{A}$ is the common information that is contained in each piece of information in the information set $\mathcal{A}$.

Below is the concept of *informalogical space* introduced in [1].

Definition 1.1. ([1]) *Let $\mathcal{S}$ be a non-empty information set, and let $\Omega = \vee\mathcal{S}$ (i.e., Ω is the union of all the information in $\mathcal{S}$). Let $\mathcal{I}$ be a non-empty subset of $\mathcal{S}$ such that $\vee\mathcal{I} = \Omega$. We say that $\mathcal{I}$ is an informalogy, that $\mathcal{S}$ is the space of the informalogy $\mathcal{I}$, that $\mathcal{I}$ is an informalogy for the space $\mathcal{S}$ and that the pair $(\mathcal{S}, \mathcal{I})$ is an informalogical space, if the following two conditions hold:*

1. *if $I, J \in \mathcal{I}$, then $I \wedge J \in \mathcal{I}$; and*
2. *if $\mathcal{I}_0 \subseteq \mathcal{I}$, then $\vee\mathcal{I}_0 \in \mathcal{I}$.*

Below are two theorems from [2] that contain some basic properties about the binary *contain* relation, and *union* and *intersection* operations.

Theorem 1.1. ([2]) *Suppose that A, B, X and Y are pieces of information, and 0 is the zero information. Then,*

1. *$A \preceq A$;*
2. *if $X \preceq A$ and $A \preceq Y$, then $X \preceq Y$;*
3. *if $A \preceq X$ and $B \preceq X$, then $A \vee B \preceq X$;*
4. *if $X \preceq A$ and $X \preceq B$, then $X \preceq A \wedge B$;*
5. *$A \wedge B \preceq A \vee B$;*
6. *$A \vee B = B \vee A$, $A \wedge B = B \wedge A$;*
7. *if $X \preceq A$ and $Y \preceq B$, then $X \vee Y \preceq A \vee B$ and $X \wedge Y \preceq A \wedge B$;*
8. *$X \vee (A \vee B) = (X \vee A) \vee B$, $X \wedge (A \wedge B) = (X \wedge A) \wedge B$; and*
9. *$A \preceq B$, $A \vee B = B$ and $A \wedge B = A$ are equivalent.*

Theorem 1.2. ([2]) *Suppose that X, A and B are pieces of information, and $\mathcal{A}$ and $\mathcal{B}$ are information sets. Then,*

1. *if $A \preceq X$ for each $A \in \mathcal{A}$, then $\vee\mathcal{A} \preceq X$; if $X \preceq A$ for each $A \in \mathcal{A}$, then $X \preceq \wedge\mathcal{A}$;*
2. *if for each $A \in \mathcal{A}$, there is $B \in \mathcal{B}$ such that $A \preceq B$, then $\vee\mathcal{A} \preceq \vee\mathcal{B}$;*
3. *$X \vee (\vee\mathcal{A}) = \vee\{X \vee A | A \in \mathcal{A}\}$, $X \wedge (\wedge\mathcal{A}) = \wedge\{X \wedge A | A \in \mathcal{A}\}$; and*
4. *$(\vee\mathcal{A}) \vee (\vee\mathcal{B}) = \vee\{A \vee B | A \in \mathcal{A}, B \in \mathcal{B}\}$, $(\wedge\mathcal{A}) \wedge (\wedge\mathcal{B}) = \wedge\{A \wedge B | A \in \mathcal{A}, B \in \mathcal{B}\}$.*

In Section 2, we generalize the concept of *closed information interval* to obtain *generalized closed information interval*. The purpose of introducing generalized closed information interval is to create a topology based on generalized closed information intervals and thus obtain a closed interval induced topological space. Based on generalized closed information interval, we introduce *generalized closed neighborhood* and *generalized closed interval cover*. Then, we redefine the concepts of *closed accumulation information* of an information set, *closed convergence* of an information net, *closed separated informalogical space*, *closed cluster information* of an information net, *closed first countable* informalogical spaces, and *closed compactness* based on generalized closed neighborhood and generalized closed interval cover instead of based on closed neighborhood and closed interval cover as in [1]. We prove that the redefined concepts based on generalized closed neighborhood and generalized closed interval cover are equivalent to the same named concepts in [1] and [2] that were based on closed neighborhood and closed interval cover. Finally, we introduce closed interval induced topology and the corresponding topological space. The closed convergence of an information net in an informalogical space is the same as the Moore-Smith style convergence of the corresponding nets in the closed interval induced topological space, and the closed compactness of the informalogical space is the same as the compactness of the closed interval induced topological space.

In Section 3, we generalize the concept of *open information interval* to obtain *generalized open information interval.* The purpose of intruding generalized open information interval is to create a topology based on generalized open information intervals and thus obtain an open interval induced topological space. Based on generalized open information interval, we introduce *generalized open neighborhood* and *generalized open interval cover*. Then, we redefine the concepts of *open accumulation information* of an information set, *open convergence* of an information net, *open separated* informalogical spaces, *open cluster information* of an information net, *open first countable* informalogical spaces and *open compactness* based on generalized open neighborhood and generalized open interval cover instead of based on open neighborhood and open interval cover as in [3]. We prove that the redefined concepts based on generalized open neighborhood and generalized open interval cover are equivalent to the same named concepts in [3] that were based on open neighborhood and open interval cover. Finally, we introduce open interval induced topology and the corresponding topological space. The open convergence of an information net in an open normal informalogical space is the same as the Moore-Smith convergence of the corresponding nets in the open interval induced topological space, and the open compactness of the open normal informalogical space is the same as the compactness of the open interval induced topological space.

With the creations of closed interval induced topological space and open interval induced topological space we build a bridge between study of informalogical spaces and study of topological spaces. The major studies we did so far in informalogical spaces, namely convergence of information nets and compactness of informalogical spaces, can be done directly as convergence of nets and compactness of topological spaces. In other words, those studies in informalogical spaces have topological nature. Because of the creations of closed interval induced topological space and open interval induced topological space we can apply existing studies and results in topological spaces directly to informalogical spaces. However, it also should be pointed out that studies in closed interval and open interval generated topological spaces cannot cover some most important characteristics of the original informalogical space, such as the containing relationship among pieces of information in the informalogical space and decompositions of pieces of information in the informalogical space. Those important characteristics of an informalogical space are lost in the topological spaces, and studies related to those characteristics have to be done in the original informalogical space.

In Section 4, we list some of our future works.

2. Generalized Closed Information Intervals and Closed Interval Induced Topological Space

In this section, we generalize the concept of *closed information interval* to obtain *generalized closed information interval*. The purpose of introducing generalized closed information interval is to create a topology based on generalized closed information intervals and thus obtain a closed interval induced topological space. Based on generalized closed information interval, we introduce *generalized closed neighborhood* and *generalized closed interval cover*. Then, we redefine the concepts of *closed accumulation information* of an information set, *closed convergence* of an information net, *closed separated informalogical space*, *closed cluster information* of an information net, *closed first countable* informalogical spaces, and *closed compactness* based on generalized closed neighborhood and generalized closed interval cover instead of based on closed neighborhood and closed interval cover as in [1]. We prove that the redefined concepts based on generalized closed neighborhood and generalized closed interval cover are equivalent to the same named concepts in [1] and [2] that were based on closed neighborhood and closed interval cover. Finally, we introduce closed interval induced topology and the corresponding topological space. The closed convergence of an information net in an informalogical space is the same as the Moore-Smith style convergence of the corresponding nets in the closed interval induced topological space, and the closed compactness of the informalogical space is the same as the compactness of the closed interval induced topological space.

Before introducing generalized closed information interval and generalized closed neighborhood, we revisit the concepts of *closed information interval* and *closed neighborhood*.

Definition 2.1. ([1]) *Let $(\mathcal{S}, \mathcal{I})$ be an informalogical space. Let X and Y be two members of the informalogy $\mathcal{I}$. We define $[X, Y]$ as $[X, Y] \equiv \{I | I \in \mathcal{S} \text{ and } X \preceq I \preceq Y\}$. $[X, Y]$ is an information set which contains all the information in $\mathcal{S}$ that ranges from the lower endpoint X to the upper endpoint Y. We call $[X, Y]$ a closed information interval in the informalogical space*

$(\mathcal{S}, \mathcal{I})$, or simply a closed interval. When $[X, Y]$ is non-empty, we call it a non-empty closed interval; when $[X, Y]$ is empty, we call it an empty closed interval, and we use θ to denote an empty closed interval.

When both $[X_1, Y_1]$ and $[X_2, Y_2]$ are closed information intervals, and $[X_1, Y_1] \subseteq [X_2, Y_2]$, we say that $[X_1, Y_1]$ is a closed subinterval of $[X_2, Y_2]$.

A set of closed intervals is called a family of closed intervals, or a closed interval family. We often use $\mathcal{U}$ to represent a closed interval family.

In cases where no confusion is likely to result, we may simply use a single letter such as U, V, etc. to represent a closed information interval. However, it should be kept in mind that a closed information interval is not a single piece of information, but a set of information.

Definition 2.2. ([1]) *Let $(\mathcal{S}, \mathcal{I})$ be an informalogical space, and let $I \in \mathcal{S}$. Let $[X, Y]$ be a non-empty closed interval in the informalogical space. If $I \in [X, Y]$, which means $X \preceq I \preceq Y$, we say that the closed interval $[X, Y]$ is a $\mathcal{I}$-closed neighborhood, or closed neighborhood for short, of I, and we use $U_{(I)}[X, Y]$ to denote this relationship. We can simply use $[X, Y]$, $U_{(I)}$ or U to denote a closed neighborhood if no confusion seems possible.*

It should be noted that closed information interval and closed neighborhood originally were simply called *information interval* and *neighborhood*, respectively, in [1]. It was after we introduced open information interval and open neighborhood in [2] did we start to call $[X, Y]$ a closed information interval and a closed neighborhood.

Now, we introduce the concepts of *generalized closed information interval* and *generalized closed neighborhood*.

Definition 2.3. *Let $(\mathcal{S}, \mathcal{I})$ be an informalogical space. We say a subset U of $\mathcal{S}$ is a generalized closed information interval, or a generalized closed interval for short, if U is a union of closed intervals. In other words, $U = \cup_{a \in A} U_a$, where A is the index set and each U_a $(a \in A)$ is a closed interval.*

Obviously, a closed interval is a generalized closed information interval since it is the union of itself. Also, similar to the case of closed information intervals, the intersection of two generalized closed information intervals is still a generalized closed information interval. This is shown in the following theorem.

Theorem 2.1. *Suppose U_1 and U_2 are two generalized closed intervals. Then, their intersection $U_1 \cap U_2$ is still a generalized closed interval.*

Proof. Let $U_1 = \cup_{a1 \in A1} U_{a1}$ and $U_2 = \cup_{a2 \in A2} U_{a2}$, where U_{a1} ($a1 \in A1$) and U_{a2} ($a2 \in A2$) are closed intervals. Then, $U_1 \cap U_2 = (\cup_{a1 \in A1} U_{a1}) \cap (\cup_{a2 \in A2} U_{a2}) = \cup\{U_{a1} \cap U_{a2} | a1 \in A1, a2 \in A2\}$. By Theorem 3.1 of [1], $U_{a1} \cap U_{a2}$ ($a1 \in A1, a2 \in A2$) is a closed interval. Thus, $U_1 \cap U_2$ is a union of closed intervals, and consequently, $U_1 \cap U_2$ is a generalized closed interval. □

Actually, by repeatedly applying Theorem 2.1, it is easy to see that the intersection of any finite number of generalized closed intervals is still a generalized closed interval.

Definition 2.4. *Let $(\mathcal{S}, \mathcal{I})$ be an informalogical space, and let I be a piece of information in the informalogical space. Let U be a subset of $\mathcal{S}$. If U contains a generalized closed interval V as a subset of which I is a member (i.e., $I \in V \subseteq U$), then we say that U is an $\mathcal{I}$-generalized closed neighborhood, or generalized closed neighborhood for short, of I.*

Definition 2.5. *We say that the family of all generalized closed neighborhoods of a piece of information I is the generalized closed neighborhood system of I, and we often use $\mathcal{U}_I$ to denote the generalized closed neighborhood system of I if no confusions seem possible.*

If $\mathcal{U}_0 \subseteq \mathcal{U}_I$, and every generalized closed neighborhood of I contains a member of $\mathcal{U}_0$ as a subset, we say that $\mathcal{U}_0$ is a generalized closed base for the generalized closed neighborhood system of I, or a generalized closed local base at I.

It should be noted that a generalized closed neighborhood need not to be a generalized closed interval but it contains a generalized closed interval as a subset, just like that a neighborhood in general topology need not to be an open set but it contains an open set (see [4, Chapter 1]). Obviously, a closed neighborhood as defined in Definition 2.2 is also a generalized closed neighborhood.

Theorem 2.2. *The intersection of any finite number of generalized closed neighborhoods of a piece of information is still a generalized closed neighborhood of that piece of information.*

PROOF. Suppose $U_1, U_2, ..., U_n \in \mathcal{U}_I$. Then, there are generalized closed intervals $V_1, V_2, ..., V_n$ such that $I \in V_i \subseteq U_i$ $(i = 1, 2, ..., n)$. Then, $I \in \cap_{i=1}^{n} V_i \subseteq \cap_{i=1}^{n} U_i$. Thus, $\cap_{i=1}^{n} U_i$ is a generalized closed neighborhood of I since $\cap_{i=1}^{n} V_i$ is still a generalized closed interval. □

In [1], based on closed neighborhood which was termed simply as neighborhood at that time, we introduced the concepts of *accumulation information* of an information set, *convergence* of an information net, *separated informalogical space*, *cluster information* an information net and *first countable* informalogical spaces. After we introduced open neighborhood in [3], to reflect the fact that those concepts in [1] were based on closed neighborhood, we renamed those concepts as *closed accumulation information*, *closed convergence*, *closed separated informalogical space*, *closed cluster information* and *closed first countable.* Now, we redefine those concepts based on generalized closed neighborhood instead of closed neighborhood. After each definition, we prove that the concept based generalized closed neighborhood is equivalent to the same named concept introduced in [1] based on closed neighborhood. Thus, for those redefined concepts, we keep the same names as closed accumulation information, closed convergence, closed separated informalogical space, closed cluster information, and closed first countable.

Definition 2.6. *Let $\mathcal{A}$ be an information set in the space. Let I be a piece of information in the space. We say that I is a piece of $\mathcal{I}$-closed accumulation information, or a piece of closed accumulation information for short, of the information set $\mathcal{A}$ if every generalized closed neighborhood of I contains a member of $\mathcal{A}$ that is different from I itself.*

We can get the definition of closed accumulation information introduced in [1] that was based on closed neighborhood when we replace the "generalized closed neighborhood" in the above definition by "closed neighborhood".

Theorem 2.3. *A piece of information I is a piece of closed accumulation information of an information set $\mathcal{A}$ based on generalized closed neighborhood if and only if I is a piece of closed accumulation information of $\mathcal{A}$ based on closed neighborhood.*

PROOF. Suppose I is a piece of closed accumulation information of $\mathcal{A}$ based on generalized closed neighborhood. Then, every generalized closed neighborhood of I contains a member of $\mathcal{A}$ that is different from I itself. Let U

be a closed neighborhood of I. Since a closed neighborhood is a generalized closed neighborhood, U contains a member of $\mathcal{A}$ that is different from I itself. Thus, I is a piece of closed accumulation information of $\mathcal{A}$ based on closed neighborhood.

Conversely, suppose I is a piece of closed accumulation information of $\mathcal{A}$ based on closed neighborhood. Then, every closed neighborhood of I contains a member of $\mathcal{A}$ that is different from I itself. Let U be a generalized closed neighborhood of I. Then, there is a generalized closed interval V such that $I \in V \subseteq U$. Since V is a generalized closed interval, V is a union of closed intervals or $V = \cup_{a \in A} V_a$, where A is the index set and each V_a is a closed interval. Since $I \in V$, there is $a0 \in A$ such that $I \in V_{a0}$. That means V_{a0} is a closed neighborhood of I. Thus, V_{a0} contains a member of $\mathcal{A}$ that is different from I itself. Consequently, U contains a member of $\mathcal{A}$ that is different from I itself since $V_{a0} \subseteq V \subseteq U$. Thus, I is a piece of closed accumulation information of $\mathcal{A}$ based on generalized closed neighborhood.
□

In [1], we introduced the concept of *information net* in an informalogical space and the concept of *closed convergence* of information nets based on closed neighborhood. We also built a Moore-Smith style convergence theory for information nets that is similar to the Moore-Smith convergence theory of nets in general topology (see [4, Chapter 2]). Before we redefine closed convergence based on generalized closed neighborhood, we revisit the concept information net in an informalogical space.

Definition 2.7. ([1]) *Let $(\mathcal{S}, \mathcal{I})$ be an informalogical space, $(D, \geq)$ be a directed set and T be a function on D whose values are pieces of information in the space. That means, for each $n \in D$, there is one and only one $T_n \in \mathcal{S}$ that corresponds to n. Then, $\{T_n, n \in D, \geq\}$ is called an information net in the space $\mathcal{S}$. In cases where no confusion would result, we simply use $\{T_n, n \in D\}$ or $\{T_n\}$ to denote an information net.*

Next, we redefine the concept of *closed convergence* of information nets based on generalized closed neighborhood.

Definition 2.8. ([3]) *Let $\{T_n, n \in D, \geq\}$ be an information net, and let $\mathcal{A}$ be an information set in an informalogical space $(\mathcal{S}, \mathcal{I})$. Then*

1. *we say that the information net $\{T_n, n \in D, \geq\}$ is in the information set $\mathcal{A}$ if $T_n \in \mathcal{A}$ for every $n \in D$;*

2. *we say that the information net* $\{T_n, n \in D, \geq\}$ *is eventually in the information set* $\mathcal{A}$ *if there is an* $m \in D$ *such that* $T_n \in \mathcal{A}$ *for every* $n \in D$ *that satisfies* $n \geq m$*; and*
3. *we say that the information net* $\{T_n, n \in D, \geq\}$ *is frequently in the information set* $\mathcal{A}$ *if, for every* $m \in D$*, there is* $n \in D$ *such that* $n \geq m$ *and* $T_n \in \mathcal{A}$*.*

Actually, we already introduced the concepts of "in", "eventually in" and "frequently in" in [1]. The only difference is that, instead of a general information set $\mathcal{A}$, we used a closed interval $[X, Y]$ at that time which confined the three concepts to only closed intervals.

Definition 2.9. *Let* $(\mathcal{S}, \mathcal{I})$ *be an informalogical space,* $\{T_n, n \in D, \geq\}$ *be an information net in the space, and* I *be a piece of information in the space. We say that the information net* $\{T_n, n \in D, \geq\}$ *closed converges to the information* I *in the informalogical space* $(\mathcal{S}, \mathcal{I})$*, or simply say that* $\{T_n, n \in D, \geq\}$ $\mathcal{I}$*-closed converges to* I*, if the information net* $\{T_n, n \in D, \geq\}$ *is eventually in every generalized closed neighborhood of* I*. The information* I *is called a piece of* $\mathcal{I}$*-closed limit information of the information net* $\{T_n, n \in D, \geq\}$ *if* $\{T_n, n \in D, \geq\}$ $\mathcal{I}$*-closed converges to* I*. When no confusion would arise, for short, we simply say that the information net* $\{T_n, n \in D, \geq\}$ *closed converges to information* I*, and that the information* I *is a piece of closed limit information of the information net* $\{T_n, n \in D, \geq\}$*.*

We can get the definition of closed convergence introduced in [1] that was based on closed neighborhood when we replace the "generalized closed neighborhood" in the above definition by "closed neighborhood".

Theorem 2.4. *An information net* $\{T_n, n \in D, \geq\}$ *closed converges to a piece of information* I *based on generalized closed neighborhood if and only if* $\{T_n, n \in D, \geq\}$ *closed converges to* I *based on closed neighborhood.*

PROOF. Suppose $\{T_n, n \in D, \geq\}$ closed converges to I based on generalized closed neighborhood. Since a closed neighborhood is a generalized closed neighborhood, $\{T_n, n \in D, \geq\}$ is eventually in every closed neighborhood of I. Thus, $\{T_n, n \in D, \geq\}$ closed converges to I based on closed neighborhood.

Conversely, suppose $\{T_n, n \in D, \geq\}$ closed converges to I based on closed neighborhood. Let U be a generalized closed neighborhood of I. Then, there is a generalized closed interval V such that $I \in V \subseteq U$. Since V is a generalized closed interval, V is a union of closed intervals or $V = \cup_{a \in A} V_a$, where A is the index set and each V_a is a closed interval. Since $I \in V$, there is $a0 \in A$ such that $I \in V_{a0}$. That means V_{a0} is a closed neighborhood of I. Thus, $\{T_n, n \in D, \geq\}$ is eventually in V_{a0}, and consequently, $\{T_n, n \in D, \geq\}$ is eventually in U, since $V_{a0} \subseteq V \subseteq U$. This shows that $\{T_n, n \in D, \geq\}$ closed converges to I based on generalized closed neighborhood. □

Definition 2.10. *Let $(\mathcal{S}, \mathcal{I})$ be an informalogical space. We say that $(\mathcal{S}, \mathcal{I})$ is a closed separated informalogical space, or say that it is closed separated, if for every two distinct pieces of information I and J in the space, i.e., $I, J \in \mathcal{S}$ and $I \neq J$, there exist generalized closed neighborhoods U and V of I and J, respectively, such that $U \cap V = \phi$.*

We can get the definition of closed separated informalogical space introduced in [1] that was based on closed neighborhood when we replace the "generalized closed neighborhood" in the above definition by "closed neighborhood".

Theorem 2.5. *An informalogical space $(\mathcal{S}, \mathcal{I})$ is a closed separated informalogical space based on generalized closed neighborhood if and only if it is a closed separated informalogical space based on closed neighborhood.*

Proof. Suppose $(\mathcal{S}, \mathcal{I})$ is a closed separated informalogical space based on generalized closed neighborhood. Let I and J be two distinct pieces of information in the space (*i.e.*, $I, J \in \mathcal{S}$ and $I \neq J$). Then, there exist generalized closed neighborhoods U and V of I and J, respectively, such that $U \cap V = \phi$.

Then, there are generalized closed intervals U' and V' such that $I \in U' \subseteq U$ and $J \in V' \subseteq V$. Since U' and V' are generalized closed intervals, U' and V' are unions of closed intervals or $U' = \cup_{a \in A} U_a$ and $V' = \cup_{b \in B} V_b$, where A and B are the index sets, and U_a and V_b are closed intervals. Since $I \in U'$ and $J \in V'$, there are $a0 \in A$ and $b0 \in B$ such that $I \in U_{a0}$ and $J \in V_{b0}$. Since $U_{a0} \subseteq U' \subseteq U$, $V_{b0} \subseteq V' \subseteq V$ and $U \cap V = \phi$, $U_{a0} \cap V_{b0} = \phi$. This shows that $(\mathcal{S}, \mathcal{I})$ is a closed separated informalogical space based on closed neighborhood.

Conversely, suppose $(\mathcal{S}, \mathcal{I})$ is a closed separated informalogical space based on closed neighborhood. Let I and J be two distinct pieces of information in the space. Then, there exist closed neighborhoods U and V of

I and J, respectively, such that $U \cap V = \phi$. Since closed neighborhoods are generalized closed neighborhoods, $(\mathcal{S}, \mathcal{I})$ is a closed separated informalogical space based on generalized closed neighborhood. $\square$

Definition 2.11. *We say that a piece of information I is a piece of closed cluster information of an information net $\{T_n\}$ if the information net $\{T_n\}$ is frequently in every generalized closed neighborhood of I.*

We can get the definition of closed cluster information introduced in [1] that was based on closed neighborhood when we replace the "generalized closed neighborhood" in the above definition by "closed neighborhood".

Theorem 2.6. *A piece of information I is a piece of closed cluster information of an information net $\{T_n\}$ based on generalized closed neighborhood if and only if I is a piece of closed cluster information of $\{T_n\}$ based on closed neighborhood*

Proof. Suppose I is a piece of closed cluster information of $\{T_n\}$ based on generalized closed neighborhood. Then, $\{T_n\}$ is frequently in every generalized closed neighborhood of I. Let U be a closed neighborhood of I. Since a closed neighborhood is a generalized closed neighborhood, $\{T_n\}$ is frequently in U. Thus, I is a piece of closed cluster information of $\{T_n\}$ based on closed neighborhood.

Conversely, suppose I is a piece of closed cluster information of $\{T_n\}$ based on closed neighborhood. Then, $\{T_n\}$ is frequently in every closed neighborhood of I. Let U be a generalized closed neighborhood of I. Then, there is a generalized closed interval V such that $I \in V \subseteq U$. Since V is a generalized closed interval, V is a union of closed intervals or $V = \cup_{a \in A} V_a$, where A is the index set and each V_a is a closed interval. Since $I \in V$, there is $a0 \in A$ such that $I \in V_{a0}$. That means V_{a0} is a closed neighborhood of I. Thus, $\{T_n\}$ is frequently in V_{a0}. Consequently, $\{T_n\}$ is frequently in U since $V_{a0} \subseteq V \subseteq U$. This means I is a piece of closed cluster information of $\{T_n\}$ based on generalized closed neighborhood. $\square$

Definition 2.12. *Let $(\mathcal{S}, \mathcal{I})$ be an informalogical space. We say that the informalogical space is closed first countable if the generalized closed neighborhood family of each piece of information in the space has a countable generalized closed base. In other words, there is a countable generalized closed local base at each piece of information in the space.*

We can get the definition of closed first countable introduced in [1] that was based on closed neighborhood when we replace the "generalized closed neighborhood" in the above definition by "closed neighborhood".

Theorem 2.7. *An informalogical space* $(\mathcal{S}, \mathcal{I})$ *is closed first countable based on generalized closed neighborhood if and only it is closed first countable based on closed neighborhood*

PROOF. Suppose $(\mathcal{S}, \mathcal{I})$ is closed first countable based on generalized closed neighborhood. Let $I \in \mathcal{S}$. Then, the generalized closed neighborhood family of I has a countable generalized closed base $U_1, U_2, ..., U_n, ...$. Since U_i $(i = 1, 2, ..., n, ...)$ is a generalized closed neighborhood of I, there is a generalized closed interval V_i such that $I \in V_i \subseteq U_i$. Since V_i is a generalized closed interval, V_i is a union of closed intervals or $V_i = \cup_{ia \in iA} V_{ia}$, where iA is the index set and each V_{ia} is a closed interval. Since $I \in V_i$, there is $ia0 \in iA$ such that $I \in V_{ia0}$. That means V_{ia0} is a closed neighborhood of I. Next, we show that $V_{1a0}, V_{2a0}, ..., V_{na0}, ...$ is a closed base for the closed neighborhood family of I.

Let W be a closed neighborhood of I. Then, W is a generalized closed neighborhood of I. Thus, there is U_i such that $U_i \subseteq W$. Then, $V_{ia0} \subseteq V_i \subseteq U_i \subseteq W$. Thus, $V_{1a0}, V_{2a0}, ..., V_{na0}, ...$ is a closed base for the closed neighborhood family of I. This shows that $(\mathcal{S}, \mathcal{I})$ is closed first countable based on closed neighborhood.

Conversely, suppose $(\mathcal{S}, \mathcal{I})$ is closed first countable based on closed neighborhood. Let $I \in \mathcal{S}$. Then, the closed neighborhood family of I has a countable closed base $U_1, U_2, ..., U_n, ...$. For each generalized closed neighborhood U of I, there is a generalized closed interval V such that $I \in V \subseteq U$. Since V is a generalized closed interval, V is a union of closed intervals or $V = \cup_{a \in A} V_a$, where A is the index set and each V_a is a closed interval. Since $I \in V$, there is $a0 \in A$ such that $I \in V_{a0}$. That means V_{a0} is a closed neighborhood of I. Thus, there is U_i such that $U_i \subseteq V_{a0}$. That implies $U_i \subseteq U$ since $V_{a0} \subseteq V \subseteq U$. This means $U_1, U_2, ..., U_n, ...$ is a countable generalized closed base for the generalized closed neighborhood family of I. This shows that $(\mathcal{S}, \mathcal{I})$ is closed first countable based on generalized closed neighborhood. □

With the establishment of equivalence of the two sets definitions for closed accumulation information, closed convergence, closed separated informalogical space, closed cluster information and closed first countable, for a same concept, we can use either of the definitions which is more convenient in a specific context. Taking closed accumulation information as an example, when we want to know whether a piece of information I is a piece of closed accumulation information of an information set $\mathcal{A}$, we only need to check whether every closed neighborhood of I contains a member of $\mathcal{A}$ that is different from I itself. When knowing that a piece of information I is a piece of closed accumulation information of an information set $\mathcal{A}$, then we can use the property that every generalized closed neighborhood of I contains a member of $\mathcal{A}$ that is different from I itself.

With the establishment of equivalence of the two sets definitions for closed accumulation information, closed convergence, closed separated informalogical space, closed cluster information and closed first countable, the theorems and lemmas we proved in [1] that were related to those concepts still hold for the redefined same named concepts above that are based on generalized closed neighborhood. Below is a list of those theorems and lemmas. One thing that needs to be noted is: in [1], closed accumulation information, closed convergence, closed separated informalogical space, closed cluster information and closed first countable were termed simply as accumulation information, convergence, separated informalogical space, cluster information and first countable.

Theorem 2.8. ([1]) *An informalogical space is closed separated if and only if every information net in the space has at most one piece of closed limit information.*

Theorem 2.9. ([1]) *A piece of information I is a piece of closed accumulation information of an information set $\mathcal{A}$ if and only if there exists an information net in $\mathcal{A} \setminus \{I\}$ that closed converges to I.*

Lemma 2.1. ([1]) *Suppose that $\{R_m, m \in E, \geq_1\}$ is an information net, I is a piece of information, and $\mathcal{U}_I$ is the generalized closed neighborhood system of I. If the information net is frequently in every member of $\mathcal{U}_I$, then, there is an information subnet of $\{R_m, m \in E, \geq_1\}$ that closed converges to I.*

Theorem 2.10. ([1]) *A piece of information I is a piece of closed cluster information of an information net $\{R_m, m \in E, \geq_1\}$ if and only if the information net has a subnet that closed converges to I.*

Theorem 2.11. ([1]) *Suppose that the informalogical space* $(\mathcal{S}, \mathcal{I})$ *is closed first countable,* I *is a piece of information in the space,* $\mathcal{A}$ *is an information set in the space, and* $\{R_m\}$ *is an information sequence in the space. Then*

1. I *is a piece of closed accumulation information of* $\mathcal{A}$ *if and only if there is an information sequence in* $\mathcal{A} \setminus \{I\}$ *that closed converges to* I*; and*
2. I *is a piece of closed cluster information of* $\{R_m\}$ *if and only if* $\{R_m\}$ *has a subsequence that closed converges to* I*.*

In [2], we introduced the concepts of *interval cover* based on intervals and introduced *compact* informalogical spaces based on interval covers. After we introduced open interval in [3], to reflect the fact that those two concepts in [2] were based on closed intervals, we renamed those concepts as *closed interval cover* and *closed compact* informalogical spaces. Now, we introduced the concept of *generalized closed interval cover* based on generalized intervals. Then, we redefine *closed compact* informalogical spaces based on generalized closed interval covers. We prove the concept of closed compact based on generalized closed interval covers is equivalent to the closed compact based on closed interval covers that was introduced in [2]. Thus, we keep the same names as *closed compact.*

Definition 2.13. *Let* $(\mathcal{S}, \mathcal{I})$ *be an informalogical space, let* $\mathcal{A}$ *be a set of information in the informalogical space (i.e.,* $\mathcal{A} \subseteq \mathcal{S}$*), and let* $\mathcal{U}$ *be a family of generalized closed intervals in the informalogical space. We say that the generalized closed interval family* $\mathcal{U}$ *is a generalized closed interval cover of the information set* $\mathcal{A}$ *if* $\mathcal{A} \subseteq \cup\mathcal{U}$*, or in other words, each piece of information in* $\mathcal{A}$ *is in some generalized closed interval in the generalized closed interval family* $\mathcal{U}$*.*

Definition 2.14. *We say that an informalogical space* $(\mathcal{S}, \mathcal{I})$ *is a closed compact informalogical space, or* $(\mathcal{S}, \mathcal{I})$ *is closed compact, if each generalized closed interval cover of the space* $\mathcal{S}$ *has a finite subcover.*

We can get the definition of *closed compact* introduced in [2] that was based on closed interval cover when we replace the "generalized closed interval cover" in the above definition by "closed interval cover".

Theorem 2.12. *An informalogical space* $(\mathcal{S}, \mathcal{I})$ *is closed compact based on generalized closed interval cover if and only it is closed compact based on closed interval cover.*

PROOF. Suppose $(\mathcal{S}, \mathcal{I})$ is closed compact based on generalized closed interval cover. A closed interval cover of $\mathcal{S}$ is also a generalized closed interval cover of $\mathcal{S}$ since each closed interval is also a generalized closed interval. Then, the closed interval cover has a finite subcover, and thus, $(\mathcal{S}, \mathcal{I})$ is closed compact based on closed interval cover.

Conversely, suppose $(\mathcal{S}, \mathcal{I})$ is closed compact based on closed interval cover. Let $\mathcal{U}$ be a generalized closed interval cover of $\mathcal{S}$. We show that $\mathcal{U}$ has a finite subcover.

Let $\mathcal{U} = \{U_k | k \in K\}$, where K is the index set. Since U_k is a generalized closed interval, it is a union of closed intervals or $U_k = \cup_{ka \in kA} U_{ka}$, where kA is the index set and each U_{ka} is a closed interval. Then, $\{U_{ka} | k \in K, ka \in kA\}$ is a closed interval cover of $\mathcal{S}$. Since $(\mathcal{S}, \mathcal{I})$ is closed compact based on closed interval cover, $\{U_{ka} | k \in K, ka \in kA\}$ has a finite subcover

$U_{k1a1}, U_{k1a2}, ..., U_{k1an1}$,
$U_{k2a1}, U_{k2a2}, ..., U_{k2an2}$,
......,
$U_{kma1}, U_{kma2}, ..., U_{kmanm}$.
Since
$U_{k1a1}, U_{k1a2}, ..., U_{k1an1} \subseteq U_{k1}$,
$U_{k2a1}, U_{k2a2}, ..., U_{k2an2} \subseteq U_{k2}$,
......,
$U_{kma1}, U_{kma2}, ..., U_{kmanm} \subseteq U_{km}$,
$\mathcal{S} \subseteq \cup_{p=1}^{m} U_{kp}$, which means that $\mathcal{U}$ has a finite subcover of $\mathcal{S}$. Thus, $(\mathcal{S}, \mathcal{I})$ is closed compact based on generalized closed interval cover. □

With the establishment of equivalence of closed compact based on generalized closed interval cover and closed interval cover, a theorem and a corollary we proved in [1] that were related to closed compact hold for the redefined same named concept based on generalized closed interval cover. Below are the theorem and corollary.

Theorem 2.13. ([2]) *An informalogical space $(\mathcal{S}, \mathcal{I})$ is closed compact if and only if each information net in the informalogical space has a piece of closed cluster information.*

Corollary 2.1. ([2]) *An informalogical space $(\mathcal{S}, \mathcal{I})$ is closed compact if and only if each information net in the informalogical space has a subnet that closed converges to a piece of information in $(\mathcal{S}, \mathcal{I})$.*

In [2], we introduced the concept of *isomorphism* between two informalogical spaces. Basically, an isomorphism between two informalogical spaces $(\mathcal{S}_1, \mathcal{I}_1)$ and $(\mathcal{S}_2, \mathcal{I}_2)$ is an order-preserving bijective function f from $(\mathcal{S}_1, \mathcal{I}_1)$ to $(\mathcal{S}_2, \mathcal{I}_2)$, and f preserves the informalogy. That is, 1) f is a bijective function from $\mathcal{S}_1$ to $\mathcal{S}_2$; 2) $f(I) \preceq f(J)$ in $\mathcal{S}_2$ if and only if $I \preceq J$ in $\mathcal{S}_1$ (order-preserving); and 3) $f(I) \in \mathcal{I}_2$ if and only if $I \in \mathcal{I}_1$ (informalogy preserving). We proved in [2] that closed convergence of information nets and closed accumulation of information sets are all preserved under isomorphisms. Since the redefined closed convergence of information nets and closed accumulation of information sets based on generalized closed neighborhood are equivalent to the same named concepts introduced in [1] that were based on closed neighborhood, results in [2] related to preservations of closed convergence and closed accumulation information hold for the redefined closed convergence and closed accumulation information based on generalized closed neighborhoods. Below are the results.

Let f be a function from $(\mathcal{S}_1, \mathcal{I}_1)$ to $(\mathcal{S}_2, \mathcal{I}_2)$. Let $\{T_n, n \in D, \geq\}$ be an information net in $(\mathcal{S}_1, \mathcal{I}_1)$. Then, $\{f(T_n), n \in D, \geq\}$ is an information net in $(\mathcal{S}_2, \mathcal{I}_2)$.

Theorem 2.14. ([2]) *Let f be an isomorphism from $(\mathcal{S}_1, \mathcal{I}_1)$ to $(\mathcal{S}_2, \mathcal{I}_2)$. The information net $\{f(T_n), n \in D, \geq\}$ closed converges to $f(T)$ in $(\mathcal{S}_2, \mathcal{I}_2)$ if and only if the information net $\{T_n, n \in D, \geq\}$ closed converges to T in $(\mathcal{S}_1, \mathcal{I}_1)$.*

Corollary 2.2. ([2]) *Let f be an isomorphism from $(\mathcal{S}_1, \mathcal{I}_1)$ to $(\mathcal{S}_2, \mathcal{I}_2)$, let $\mathcal{A}$ be an information set in $(\mathcal{S}_1, \mathcal{I}_1)$ and let I be a piece of information in $(\mathcal{S}_1, \mathcal{I}_1)$. Then, $f(I)$ is a closed accumulation information of the information set $f(\mathcal{A})$ in $(\mathcal{S}_2, \mathcal{I}_2)$ if and only if I is a closed accumulation information of the information set $\mathcal{A}$ in $(\mathcal{S}_1, \mathcal{I}_1)$.*

In [2], we also introduced the concept of *isomorphic invariant* which is a property of an informalogical space that is preserved under isomorphisms. We proved in [2] that closed separatedness, closed limit uniqueness, closed first countability and closed compactness are all isomorphic invariants. Since the redefined closed separatedness, closed convergence, closed first countability and closed compactness are equivalent to same named concepts we introduced in [1] and [2], those redefined properties are also isomorphic invariants.

In the above, we introduce the *generalized closed intervals* and redefine various concepts that we introduced in [1] and [2] based on generalized closed intervals (and thus generalized closed neighborhoods and generalized closed interval covers). We prove the equivalence of the redefined concepts with the original same named concepts in [1] and [2]. The purpose that we introduce generalized closed intervals and redefine various concepts based on generalized closed intervals is for the introduction of a topology and a corresponding topological space. Next, based on generalized closed intervals, we introduce a topology for the *space* $\mathcal{S}$ of an informalogical space $(\mathcal{S}, \mathcal{I})$. In this paper, we adopt the definitions in [4] for all concepts in general topology.

Definition 2.15. *Let $(\mathcal{S}, \mathcal{I})$ be an informalogical space. Let $\mathcal{T}_c$ be the family of all generalized closed intervals in the informalogical space $(\mathcal{S}, \mathcal{I})$. Then, $\mathcal{T}_c$ is a topology for $\mathcal{S}$. We call this topology closed interval induced topology in the informalogical space $(\mathcal{S}, \mathcal{I})$, and we call $(\mathcal{S}, \mathcal{T}_c)$ closed interval induced topological space in the informalogical space $(\mathcal{S}, \mathcal{I})$. When no confusions seem possible, we can simply call $\mathcal{T}_c$ a closed interval induced topology and call $(\mathcal{S}, \mathcal{T}_c)$ a closed interval induced topological space.*

We show that $\mathcal{T}_c$ in the above definition is really a topology. For a subfamily $\mathcal{T}_0$ of $\mathcal{T}_c$, since each member of $\mathcal{T}_0$ is a generalized closed interval which means each member of $\mathcal{T}_0$ is a union of closed intervals, the union $\cup\mathcal{T}_0$ of $\mathcal{T}_0$ is still a union of closed intervals. That means $\cup\mathcal{T}_0$ is a generalized closed interval. Thus, $\cup\mathcal{T}_0 \in \mathcal{T}_c$. For $U, V \in \mathcal{T}_c$, since U and V are two generalized closed intervals, we have $U = \cup_{a\in A} U_a$ and $V = \cup_{b\in B} V_b$, where A and B are the index sets, and U_a, V_b are closed intervals. Then, $U \cap V = (\cup_{a\in A} U_a) \cap (\cup_{b\in B} V_b) = \cup_{a\in A, b\in B}(U_a \cap V_b)$. By Theorem 3.1 of [1], $U_a \cap V_b$ is a closed interval. Thus, $U \cap V$ is a generalized closed interval and consequently, $U \cap V \in \mathcal{T}_c$. Now, we know $\mathcal{T}_c$ is really a topology. It is obvious $\phi, \mathcal{S} \in \mathcal{T}_c$ since $[\Omega, 0] = \phi$ and $[0, \Omega] = \mathcal{S}$ are two closed intervals. We use letter "c" in $\mathcal{T}_c$ to indicate that the topology is a closed interval induced topology. That is meant to differentiate $\mathcal{T}_c$ from an open interval induced topology to be introduced in the next section of this paper.

Once the closed interval induced topological space $(\mathcal{S}, \mathcal{T}_c)$ is introduced we can examine the relationship between the topological space $(\mathcal{S}, \mathcal{T}_c)$ and the informalogical space $(\mathcal{S}, \mathcal{I})$ from which the topological space is derived. An *open set* in the topological space is a generalized closed interval in the informalogical space, an *open cover* of $\mathcal{S}$ in the topological space is a generalized closed interval cover of $\mathcal{S}$ in the informalogical space, and a *neighborhood* of a point I (actually, a piece of information) in the topological space is a generalized closed neighborhood of I in the informalogical space. A *net* in the topological space is an information net in the informalogical space. Once this is clear it is obvious that the various concepts in an informalogical space that are redefined in this paper using generalized closed intervals, generalized closed neighborhoods and generalized closed interval covers are exactly the same as corresponding concepts in a topological space. Those concepts include closed accumulation information of an information set, closed convergence of information nets, closed separatedness, closed cluster information of an information net, closed first countability and closed compactness. The following table shows the correspondence between those concepts in an informalogical space and corresponding concepts in a topological space.

Informalogical space $(\mathcal{S}, \mathcal{I})$	Topological space $(\mathcal{S}, \mathcal{T}_c)$
closed accumulation information	accumulation point ([4, p. 41])
closed convergence	Moore-Smith convergence ([4, p. 66])
closed separated	Hausdorff (or, separated) ([4, p. 67])
closed cluster information	cluster point ([4, p. 71])
closed first countability	first countability ([4, p. 50])
closed compactness	compactness ([4, p. 135])

Now, the purpose of introducing generalized closed intervals and redefining various concepts in this paper becomes clear: It is for building a bridge between informalogical spaces and topological spaces. It is also clear now that all the theorems in [1] and [2] that are related to the concepts in the above table can be obtained directly from the corresponding existing results in general topology once the bridge between informalogical spaces and topological spaces is built. This also shows that the concepts informalogical spaces in the above table have topological nature. With the bridge between informalogical spaces and topological spaces being built, we can apply more existing concepts and results in general topology directly to the study of informalogical spaces and thus try apply existing concepts and results in general topology to real world applications in information sciences. These fall into our future works.

With the usefulness of building the bridge between an informalogical space $(\mathcal{S}, \mathcal{I})$ and a topological spaces $(\mathcal{S}, \mathcal{T}_c)$ being obvious, it is also worthy to point out the limitations of study in only the topological space $(\mathcal{S}, \mathcal{T}_c)$. The topological space $(\mathcal{S}, \mathcal{T}_c)$ does not cover all characteristics of the original informalogical space $(\mathcal{S}, \mathcal{I})$. One example is the relationship among information in $\mathcal{S}$ such as one piece of information contains another piece of information. Another example is the *decompositions* of a piece of information and of an information set (see [2]). Decompositions of information and an informalogical space is an important subject. Decompositions of information in the informalogical space $(\mathcal{S}, \mathcal{I})$ will be useful to transfer the study of the original information in the informalogical space $(\mathcal{S}, \mathcal{I})$ to study of more basic pieces of information. Decompositions of information are also important in actually building useful informalogical spaces. In summary, there are important subjects in informalogical spaces that are not covered by study of only the corresponding closed interval induced topological spaces.

3. Generalized Open Information Intervals and Open Interval Induced Topological Space

In [3] we showed an undesirable property of closed convergence of information nets in an informalogical space $(\mathcal{S}, \mathcal{I})$: for an information net to closed converge to a piece of information I in the informalogy (*i.e.*, $I \in \mathcal{I}$), it is necessary that the information net be eventually identical to I. Since the closed convergence of an information net in $(\mathcal{S}, \mathcal{I})$ is the same as the convergence of a net in the closed interval induced topological space $(\mathcal{S}, \mathcal{T}_c)$, in the closed interval induced topological space $(\mathcal{S}, \mathcal{T}_c)$, the necessary condition for a net to converge to a point I in $\mathcal{I}$) is that the net be eventually identical to I. This is also obvious by examining the topological space $(\mathcal{S}, \mathcal{T}_c)$ directly.

For $I \in \mathcal{I}$, $\{I\} = [I, I]$ is a closed interval in $(\mathcal{S}, \mathcal{I})$. Thus, the singleton $\{I\}$ is an open set in $(\mathcal{S}, \mathcal{T}_c)$ (*i.e.*, $\{I\} \in \mathcal{T}_c$). Consequently, $\{I\}$ is a neighborhood of I in the topological space $(\mathcal{S}, \mathcal{T}_c)$. The consequence of this fact is that a net needs to be eventually in $\{I\}$ if the net is to converge to I in $(\mathcal{S}, \mathcal{T}_c)$. In other words, the net needs to be eventually identical to $\{I\}$ if the net is to converge to I.

To avoid the above undesirable property of closed convergence in an informalogical space, [3] introduced the concept of *open information intervals* in informalogical spaces and thus introduced *open convergence* of information nets in an informalogical space. The open convergence avoids the above undesirable property of closed convergence. However, [3] also pointed out the limitation of open convergence: for open convergence, a Moore-Smith style convergence theory cannot be established for general informalogical spaces, and the convergence theory can only be established in *open normal* informalogical spaces. An open normal informalogical space is an informalogical space in which the intersection of any two open intervals is still an open interval.

In this section, we generalize the concept of *open information interval* to obtain *generalized open information interval.* The purpose of introducing generalized open information interval is to create a topology based on generalized open information intervals and thus obtain an open interval induced topological space. Based on generalized open information interval, we introduce *generalized open neighborhood* and *generalized open interval cover.* Then, we redefine the concepts of *open accumulation information* of an information set, *open convergence* of an information net, *open separated* informalogical spaces, *open cluster information* of an information net, *open first countable* informalogical spaces and *open compactness* based on generalized open neighborhood and generalized open interval cover instead of based on open neighborhood and open interval cover as in [3]. We prove that the redefined concepts based on generalized open neighborhood and generalized open interval cover are equivalent to the same named concepts in [3] that were based on open neighborhood and open interval cover. Finally, we introduce open interval induced topology and the corresponding topological space.

Before introducing generalized open information interval, generalized open neighborhood and generalized open interval cover, we revisit the concepts of *open information interval*, or *open interval* for short, *open neighborhood* and *open interval cover.*

Definition 3.1. ([3]) *Let* $(\mathcal{S}, \mathcal{I})$ *be an informalogical space. Let* X *and* Y *be two members of the informalogy* $\mathcal{I}$*. We define* (X, Y) *as* $(X, Y) \equiv \{I | I \in \mathcal{S} \text{ and } X \prec I \prec Y\}$*.* (X, Y) *is an information set which contains all the information in* $\mathcal{S}$ *that ranges from the lower endpoint* X *to the upper endpoint* Y *and that is not* X *or* Y*. We call* (X, Y) *a proper open information interval in the informalogical space* $(\mathcal{S}, \mathcal{I})$*, or simply a proper open interval.*

We define $[0, Y)$ *as* $[0, Y) \equiv \{I | I \in \mathcal{S} \text{ and } 0 \preceq I \prec Y\}$*, and we define* $(X, \Omega]$ *as* $(X, \Omega] \equiv \{I | I \in \mathcal{S} \text{ and } X \prec I \preceq \Omega\}$*. We call* $[0, Y)$ *a lower special open information interval in the informalogical space* $(\mathcal{S}, \mathcal{I})$*, or simply a lower special open interval, and we call* $(X, \Omega]$ *an upper special open information interval in the informalogical space* $(\mathcal{S}, \mathcal{I})$*, or simply an upper special open interval.*

The aggregation of proper open information intervals and special open information intervals are called open information intervals, or simply open *intervals. Thus, an open interval can be either a proper open interval or a special open interval. We can use a single letter, such as U, to represent an open interval. However, it should be kept in mind that an open interval is an information set, not a single piece of information.*

When both U and V are open intervals (proper open intervals), and $U \subseteq V$, we say that U is an open subinterval (proper open subinterval) of V.

A set of open (proper open) intervals is called a family of open (proper open) intervals, or an open (proper open) interval family.

As mentioned in [3], proper open intervals are the natural and important open intervals. Thus, for any concepts based on open intervals, [3] also introduced parallel concepts based on proper open intervals. This can be seen in the definitions below.

Definition 3.2. ([3]) *Let $(\mathcal{S}, \mathcal{I})$ be an informalogical space, and let I be a piece of information (proper information) in the informalogical space. Let U be a non-empty open (proper open) interval in the informalogical space. When $I \in U$, we say that U is an $\mathcal{I}$-open ($\mathcal{I}$-proper open) neighborhood, or open (proper open) neighborhood for short, of I. We can use $U_{(I)}$ or simply U to denote an open (proper open) neighborhood if no confusion seems possible.*

Definition 3.3. ([3]) *We say that the family of all open (proper open) neighborhoods of a piece of information (proper information) I is the open (proper open) neighborhood system of I. When no confusion seems possible, we often use $\mathcal{U}_I$ to denote the open (proper open) neighborhood system of I.*

If $\mathcal{U}_0 \subseteq \mathcal{U}_I$, and every open (proper open) neighborhood of I contains a member of $\mathcal{U}_0$ as an open (proper open) subinterval, we say that $\mathcal{U}_0$ is an open (proper open) base for the open (proper open) neighborhood system of I, or an open (proper open) local base at I.

Definition 3.4. ([3]) *Let $(\mathcal{S}, \mathcal{I})$ be an informalogical space, let $\mathcal{A}$ be a set of information (proper information) in the informalogical space (i.e., $\mathcal{A} \subseteq \mathcal{S}$), and let $\mathcal{U}$ be a family of open (proper open) intervals in the informalogical space. We say that the open (proper open) interval family $\mathcal{U}$ is an open (proper open) interval cover of the information (proper information) set $\mathcal{A}$ if $\mathcal{A} \subseteq \cup\mathcal{U}$, or in other words, each piece of information (proper information) in $\mathcal{A}$ is in some open (proper open) interval in the open (proper open) interval family $\mathcal{U}$.*

The expression "... open (proper open) ..." in the above definitions means that we can replace "open" by "proper open" to get parallel definitions for the proper open case. We use this type of expressions in various definitions and theorems hereafter in this paper too. All of the expressions carry similar meanings and we do not explain in each occasion. The use of the expression "... open (proper open) ..." in definitions and theorems can establish parallel and independent definitions and theorems for both the open and the proper open case, and at the same time, the use of the expression makes the definitions and theorems concise.

Now, we introduce the concepts of *generalized open information interval*, *generalized open neighborhood* and *generalized open interval cover*.

Definition 3.5. *Let* $(\mathcal{S}, \mathcal{I})$ *be an informalogical space. We say a subset* U *of* $\mathcal{S}$ *is a generalized open (proper open) information interval, or a generalized open (proper open) interval for short, if* U *is a union of open (proper open) intervals. In other words,* $U = \cup_{a \in A} U_a$, *where* A *is the index set and each* U_a *is an open (proper open) interval.*

Obviously, an open interval is a generalized open interval since it is the union of itself.

Definition 3.6. *Let* $(\mathcal{S}, \mathcal{I})$ *be an informalogical space, and let* I *be a piece of information (proper information) in the informalogical space. Let* U *be a subset of* $\mathcal{S}$. *If* U *contains a generalized open (proper open) interval* V *as a subset of which* I *is a member (i.e.,* $I \in V \subseteq U$*), then we say that* U *is an* $\mathcal{I}$*-generalized open (proper open) neighborhood, or generalized open (proper open) neighborhood for short, of* I.

Definition 3.7. *We say that the family of all generalized open (proper open) neighborhoods of a piece of information (proper information)* I *is the generalized open (proper open) neighborhood system of* I, *and we often use* $\mathcal{U}_I$ *to denote the generalized open (proper open) neighborhood system of* I *if no confusions seem possible.*

If $\mathcal{U}_0 \subseteq \mathcal{U}_I$, *and every generalized open (proper open) neighborhood of* I *contains a member of* $\mathcal{U}_0$ *as a subset, we say that* $\mathcal{U}_0$ *is a generalized open (proper open) base for the generalized open neighborhood system of* I, *or a generalized open (proper open) local base at* I.

Definition 3.8. *Let* $(\mathcal{S}, \mathcal{I})$ *be an informalogical space, let* $\mathcal{A}$ *be a set of information (proper information) in the informalogical space (i.e.,* $\mathcal{A} \subseteq \mathcal{S}$*), and let* $\mathcal{U}$ *be a family of generalized open (proper open) intervals in the informalogical space. We say that the generalized open (proper open) interval family* $\mathcal{U}$ *is a generalized open (proper open) interval cover of the information (proper information) set* $\mathcal{A}$ *if* $\mathcal{A} \subseteq \cup\mathcal{U}$*, or in other words, each piece of information (proper information) in* $\mathcal{A}$ *is in some generalized open (proper open) interval in the generalized open (proper open) interval family* $\mathcal{U}$*.*

In [3], based on open neighborhood and open interval cover, we introduced the concepts of *open accumulation information*, *open convergence*, *open separated* informalogical spaces, *open cluster information*, *open first countable* informalogical spaces and *open compact* informalogical spaces. Now, we redefine those concepts based on generalized open neighborhood and generalized open interval cover instead of open neighborhood and open interval cover. After each definition, we prove that the concept based on generalized open neighborhood, or generalized open interval cover in the case of open compactness, is equivalent to the same named concept based open neighborhood, or open interval cover in the case of open compactness, introduced in [3]. Thus, we keep the same names as open accumulation information, open convergence, open separated informalogical spaces, open cluster information, open first countable informalogical spaces and open compact informalogical spaces.

Definition 3.9. *Let* $\mathcal{A}$ *be an information set in an informalogical space* $(\mathcal{S}, \mathcal{I})$ *(i.e.,* $\mathcal{A} \subseteq \mathcal{S}$*). Let* I *be a piece of information (proper information) in the informalogical space (i.e.,* $I \in \mathcal{S}$*). We say that* I *is a piece of* $\mathcal{I}$*-open (*$\mathcal{I}$*-proper open) accumulation information, or a piece of open (proper open) accumulation information for short, of the information set* $\mathcal{A}$ *if every generalized open (proper open) neighborhood of* I *contains a member of* $\mathcal{A}$ *that is different from* I *itself.*

We can get the definition of open accumulation information introduced in [3] that was based on open neighborhood when we replace the "generalized open (proper open) neighborhood" in the above definition by "open (proper open) neighborhood".

Theorem 3.1. *A piece of information* I *is a piece of open (proper open) accumulation information of an information set* $\mathcal{A}$ *based on generalized open (proper open) neighborhood if and only if* I *is a piece of open (proper open) accumulation information of* $\mathcal{A}$ *based on open (proper open) neighborhood*

Proof of Theorem 3.1 is identical to the proof of Theorem 2.3 except change of "closed" to "open".

Definition 3.10. *Let $(\mathcal{S}, \mathcal{I})$ be an informalogical space, $\{T_n, n \in D, \geq\}$ be an information net in the space, and I be a piece of information (proper information) in the space. We say that the information net $\{T_n, n \in D, \geq\}$ open (proper open) converges to I in the informalogical space $(\mathcal{S}, \mathcal{I})$, or say that $\{T_n, n \in D, \geq\}$ $\mathcal{I}$-open ($\mathcal{I}$-proper open) converges to I, if the information net $\{T_n, n \in D, \geq\}$ is eventually in every generalized open (proper open) neighborhood of I. The information I is called a piece of $\mathcal{I}$-open ($\mathcal{I}$-proper open) limit information of the information net $\{T_n, n \in D, \geq\}$ if $\{T_n, n \in D, \geq\}$ $\mathcal{I}$-open ($\mathcal{I}$-proper open) converges to I. When no confusion would arise, for short, we simply say that the information net $\{T_n, n \in D, \geq\}$ open (proper open) converges to information I, and that the information I is a piece of open (proper open) limit information of the information net $\{T_n, n \in D, \geq\}$.*

We can get the definition of open convergence introduced in [3] that was based on open neighborhood when we replace the "generalized open (proper open) neighborhood" in the above definition by "open (proper open) neighborhood".

Theorem 3.2. *An information net $\{T_n, n \in D, \geq\}$ open (proper open) converges to a piece of information I based on generalized open (proper open) neighborhood if and only if $\{T_n, n \in D, \geq\}$ open (proper open) converges to I based on open (proper open) neighborhood.*

Proof of Theorem 3.2 is identical to the proof of Theorem 2.4 except change of "closed" to "open".

Definition 3.11. *Let $(\mathcal{S}, \mathcal{I})$ be an informalogical space. We say that $(\mathcal{S}, \mathcal{I})$ is an open (proper open) separated informalogical space, or say that it is open (proper open) separated, if for every two distinct pieces of information (proper information) I and J in the space, i.e., $I, J \in \mathcal{S}$ and $I \neq J$, there exist generalized open (proper open) neighborhoods U and V of I and J, respectively, such that $U \cap V = \phi$.*

We can get the definition of open separated informalogical space introduced in [3] that was based on open neighborhood when we replace the "generalized open (proper open) neighborhood" in the above definition by "open (proper open) neighborhood".

Theorem 3.3. *An informalogical space $(\mathcal{S}, \mathcal{I})$ is an open (proper open) separated informalogical space based on generalized open (proper open) neighborhood if and only if it is an open (proper open) separated informalogical space based on open (proper open) neighborhood.*

Proof of Theorem 3.3 is identical to the proof of Theorem 2.5 except change of "closed" to "open".

Definition 3.12. *We say that a piece of information (proper information) I is a piece of open (proper open) cluster information of an information net $\{T_n, n \in D, \geq\}$ if the information net $\{T_n, n \in D, \geq\}$ is frequently in every generalized open (proper open) neighborhood of I.*

We can get the definition of open cluster information introduced in [3] that was based on open neighborhood when we replace the "generalized open (proper open) neighborhood" in the above definition by "open (proper open) neighborhood".

Theorem 3.4. *A piece of information I is a piece of open (proper open) cluster information of an information net $\{T_n\}$ based on generalized open (proper open) neighborhood if and only if I is a piece of open (proper open) cluster information of $\{T_n\}$ based on open (proper open) neighborhood*

Proof of Theorem 3.4 is identical to the proof of Theorem 2.6 except change of "closed" to "open".

Definition 3.13. *Let $(\mathcal{S}, \mathcal{I})$ be an informalogical space. We say that the informalogical space is open (proper open) first countable if the generalized open (proper open) neighborhood family of each piece of information (proper information) in the space has a countable generalized open (proper open) base. In other words, there is a countable generalized open (proper open) local base at each piece of information (proper information) in the space.*

We can get the definition of open first countable introduced in [3] that was based on open neighborhood when we replace the "generalized open (proper open) neighborhood" in the above definition by "open (proper open) neighborhood".

Theorem 3.5. *An informalogical space* $(\mathcal{S}, \mathcal{I})$ *is open (proper open) first countable based on generalized open (proper open) neighborhood if and only it is open (proper open) first countable based on open (proper open) neighborhood*

Proof of Theorem 3.5 is identical to the proof of Theorem 2.7 except change of "closed" to "open".

Definition 3.14. *We say that an informalogical space* $(\mathcal{S}, \mathcal{I})$ *is an open (proper open) compact informalogical space, or* $(\mathcal{S}, \mathcal{I})$ *is open (proper open) compact, if each generalized open (proper open) interval cover* $\mathcal{U}$ *of* $\mathcal{S}$ *(*$\mathcal{S} \setminus \{0, \Omega\}$*) has a finite generalized open (proper open) subcover, or in other words, there are finite members* U_1, U_2, ..., U_n *of* $\mathcal{U}$ *such that* $\mathcal{S} \subseteq \cup_{i=1}^{n} U_i$ *(* $\mathcal{S} \setminus \{0, \Omega\} \subseteq \cup_{i=1}^{n} U_i$ *).*

We can get the definition of *open compact* introduced in [2] that was based on open interval cover when we replace the "generalized open (proper open) interval cover" in the above definition by "open (proper open) interval cover".

Theorem 3.6. *An informalogical space* $(\mathcal{S}, \mathcal{I})$ *is open (proper open) compact based on generalized open (proper open) interval cover if and only it is open (proper open) compact based on open (proper open) interval cover.*

Proof of Theorem 3.6 is identical to the proof of Theorem 2.12 except change of "closed" to "open".

With the establishment of equivalence of the two sets of definitions for open accumulation information, open convergence, open separated informalogical spaces, open cluster information, open first countable informalogical spaces, and open compact informalogical spaces, the theorems and lemmas we proved in [3] that were related to those concepts still hold for the redefined same named concepts above that are based on generalized open neighborhood and generalized open interval cover. Before we list the results from [3] we first revisit the concept of *open normal* informalogical space from [3] since the concept is contained in many results from [3].

Definition 3.15. ([3]) *Let* $(\mathcal{S}, \mathcal{I})$ *be an informalogical space. We say that* $(\mathcal{S}, \mathcal{I})$ *is an open (proper open) normal informalogical space if the intersection of any two open (proper open) intervals is still an open (proper open) interval.*

As having been mentioned in [3], the advantage of open convergence of information nets is that it avoids the undesirable property of closed convergence of information nets described at the beginning of this section, but the disadvantage of open convergence is that a Moore-Smith convergence theory cannot be established for open convergence in general informalogical spaces. The Moore-Smith convergence theory can only be established for open convergence in open normal informalogical spaces. This is reflected in the results from [3] listed below which are still valid for the various redefined concepts based on generalized open neighborhood and generalized open interval cover.

Theorem 3.7. ([3]) *Suppose informalogical space $(\mathcal{S},\mathcal{I})$ is an open (proper open) normal informalogical space. Then, $(\mathcal{S},\mathcal{I})$ is open (proper open) separated if and only if every information net in the space has at most one piece of open (proper open) limit information.*

Theorem 3.8. ([3]) *Suppose informalogical space $(\mathcal{S},\mathcal{I})$ is an open (proper open) normal informalogical space. Then, a piece of information (proper information) I is a piece of open (proper open) accumulation information of an information set $\mathcal{A}$ if and only if there exists an information net in $\mathcal{A}\setminus\{I\}$ that open (proper open) converges to I.*

Lemma 3.1. ([3]) *Let $(\mathcal{S},\mathcal{I})$ be an open (proper open) normal informalogical space. Suppose that $\{R_m, m \in E, \geq_1\}$ is an information net in the space, I is a piece of information (proper information) in the space, and $\mathcal{U}_I$ is the open (proper open) neighborhood system of I. If the information net is frequently in every member of $\mathcal{U}_I$, then, there is an information subnet of $\{R_m, m \in E, \geq_1\}$ that open (proper open) converges to I.*

Note that Lemma 3.1 is still valid if we change "open (proper open) neighborhood system" to "generalized open (proper open) neighborhood system".

Theorem 3.9. ([3]) *Let $(\mathcal{S},\mathcal{I})$ be an open (proper open) normal informalogical space. Then, a piece of information (proper information) I is a piece of open (proper open) cluster information of an information net $\{R_m, m \in E, \geq_1\}$ if and only if the information net has a subnet that open (proper open) converges to I.*

Theorem 3.10. ([3]) *Let $(\mathcal{S},\mathcal{I})$ be an open (proper open) normal informalogical space. Suppose that the informalogical space $(\mathcal{S},\mathcal{I})$ is open (proper* *open) first countable, I is a piece of information (proper information) in the space, $\mathcal{A}$ is an information set in the space, and $\{R_m\}$ is an information sequence in the space. Then*

1. *I is a piece of open (proper open) accumulation information of $\mathcal{A}$ if and only if there is an information sequence in $\mathcal{A} \setminus \{I\}$ that open (proper open) converges to I; and*
2. *I is a piece of open (proper open) cluster information of $\{R_m\}$ if and only if $\{R_m\}$ has a subsequence that open (proper open) converges to I.*

Theorem 3.11. ([3]) *An informalogical space $(\mathcal{S}, \mathcal{I})$ is open (proper open) compact if and only if each information net in the informalogical space has a piece of open (proper open) cluster information.*

Corollary 3.1. ([3]) *Let $(\mathcal{S}, \mathcal{I})$ be an open (proper open) normal informalogical space. Then, $(\mathcal{S}, \mathcal{I})$ is open (proper open) compact if and only if each information net in the informalogical space has a subnet that open (proper open) converges to a piece of information (proper information) in $(\mathcal{S}, \mathcal{I})$.*

Also, same as in [3], for the various redefined concepts based on generalized open neighborhood and generalized open interval cover, it is still valid that open limit uniqueness, open separatedness, open first countability and open compactness are all isomorphic invariants.

In an open normal Informalogical space, the intersection of two generalized open information intervals is still a generalized open information interval. This property is critical in our introduction of a topology based on generalized open intervals, and this property is shown in the following theorem.

Theorem 3.12. *Suppose $(\mathcal{S}, \mathcal{I})$ is an open (proper open) normal informalogical space, and suppose U_1 and U_2 are two generalized open (proper open) intervals in the informalogical space. Then, their intersection $U_1 \cap U_2$ is still a generalized open (proper open) interval.*

Proof. We only show proof of the open case. Proof of the proper open case is similar.

Let $U_1 = \cup_{a1 \in A1} U_{a1}$ and $U_2 = \cup_{a2 \in A2} U_{a2}$, where $A1$ and $A2$ are the index sets, and U_{a1} and U_{a2} are open intervals. Then, $U_1 \cap U_2 = (\cup_{a1 \in A1} U_{a1}) \cap (\cup_{a2 \in A2} U_{a2}) = \cup \{U_{a1} \cap U_{a2} | a1 \in A1, a2 \in A2\}$. Since $(\mathcal{S}, \mathcal{I})$ is open normal, $U_{a1} \cap U_{a2}$ $(a1 \in A1, a2 \in A2)$ is an open interval. Thus, $U_1 \cap U_2$ is a union of open intervals, and consequently, $U_1 \cap U_2$ is a generalized open interval. □

In fact, by repeatedly applying Theorem 3.12, it is easy to see that, in an open normal informalogical space, the intersection of any finite number of generalized open intervals is still a generalized open interval. We use this fact later on without explicit reference.

Theorem 3.13. *In an open normal informalogical space, the intersection of any finite number of generalized open (proper open) neighborhoods of a piece of information is still a generalized open (proper open) neighborhood of that piece of information.*

PROOF. We only show proof of the open case. Proof of the proper open case is similar.

Suppose $U_1, U_2, ..., U_n$ are generalized open neighborhoods of a piece of information I. Then, there are generalized open intervals $V_1, V_2, ..., V_n$ such that $I \in V_i \subseteq U_i$ $(i = 1, 2, ..., n)$. Then, $I \in \cap_{i=1}^{n} V_i \subseteq \cap_{i=1}^{n} U_i$. Since the informalogical space is open normal, by Theorem 3.12, $\cap_{i=1}^{n} V_i$ is a generalized open interval. Thus, $\cap_{i=1}^{n} U_i$ is a generalized open neighborhood of I. $\square$

With all the preparations done we can now introduce a topology in an informalogical space based on generalized open intervals. The topology is called *open interval induced topology.* Also, it should be pointed out that the open interval induced topology cannot be established in general informalogical spaces. The open interval induced topology can only be established in open normal informalogical spaces.

Definition 3.16. *Let $(\mathcal{S}, \mathcal{I})$ be an open (proper open) normal informalogical space. Let $\mathcal{T}_o$ be the family of all generalized open (proper open) intervals in the informalogical space $(\mathcal{S}, \mathcal{I})$. Then, $\mathcal{T}_o$ is a topology for $\mathcal{S}$ $(\mathcal{S} \setminus \{0, \Omega\})$. We call this topology open (proper open) interval induced topology in the informalogical space $(\mathcal{S}, \mathcal{I})$, and we call $(\mathcal{S}, \mathcal{T}_o)$ $((\mathcal{S} \setminus \{0, \Omega\}, \mathcal{T}_o))$ open (proper open) interval induced topological space in the informalogical space $(\mathcal{S}, \mathcal{I})$. When no confusions seem possible, we can simply call $\mathcal{T}_o$ an open (proper open) interval induced topology and call $(\mathcal{S}, \mathcal{T}_o)$ $((\mathcal{S} \setminus \{0, \Omega\}, \mathcal{T}_o))$ an open (proper open) interval induced topological space.*

We show that $\mathcal{T}_o$ in the above definition is really a topology. We only show proof of the open case. Proof of the proper open case is similar.

For a subfamily $\mathcal{T}_0$ of $\mathcal{T}_o$, since each member of $\mathcal{T}_0$ is a generalized open interval and thus is a union of open intervals, the union $\cup \mathcal{T}_0$ of $\mathcal{T}_0$ is still a union of open intervals. That means $\cup\mathcal{T}_0$ is a generalized open interval. Thus, $\cup\mathcal{T}_0 \in \mathcal{T}_o$. For $U, V \in \mathcal{T}_o$, since U and V are two generalized open intervals, we have $U = \cup_{a\in A}U_a$ and $V = \cup_{b\in B}V_b$, where A and B are the index sets, and U_a, V_b are open intervals. Then, $U \cap V = (\cup_{a\in A}U_a) \cap (\cup_{b\in B}V_b) = \cup_{a\in A,b\in B}(U_a \cap V_b)$. Since $(\mathcal{S}, \mathcal{I})$ is open normal $U_a \cap V_b$ is an open interval. Thus, $U \cap V$ is a generalized open interval and consequently, $U \cap V \in \mathcal{T}_o$. Now, we know $\mathcal{T}_o$ is really a topology. It is obvious $\phi, \mathcal{S} \in \mathcal{T}_o$ since $(\Omega, 0) = \phi$ and $[0, \Omega) \cup (0, \Omega] = \mathcal{S}$ are two generalized open intervals. We use the letter "o" in $\mathcal{T}_o$ to indicate that the topology is an open interval induced topology which is meant to differentiate it from a closed interval induced topology.

Definition 3.16 actually introduces two topological spaces. One is the open interval induced topological space $(\mathcal{S}, \mathcal{T}_o)$ and the other is the proper open interval induced topological space $(\mathcal{S} \setminus \{0, \Omega\}, \mathcal{T}_o)$. As mentioned in [3], the reason for including special open intervals (*i.e.*, lower and upper special open intervals) in open intervals in addition to proper open intervals is that otherwise there would be no open intervals that can contain the two special (and also trivial) pieces of information 0 and Ω, and consequently, 0 and Ω would not be able to have open neighborhoods in the informalogical space $(\mathcal{S}, \mathcal{I})$. Then, it would be theoretically impossible for any information net to converge to either 0 or Ω. That would look conceptually bad even though it may not be practically bad. The inclusion of special open intervals in open intervals at least avoids that conceptual drawback. For the proper open interval induced topological space $(\mathcal{S} \setminus \{0, \Omega\}, \mathcal{T}_o)$, there is no such a conceptual issue since the space $\mathcal{S} \setminus \{0, \Omega\}$ does not contain the two special (and also trivial) pieces of information 0 and Ω.

When we discuss relationship between informalogical spaces and topological spaces below, we only discuss open interval induced topological spaces since the case of proper open interval induced topological spaces are similar.

Once the open interval induced topological space $(\mathcal{S}, \mathcal{T}_o)$ is introduced we can examine the relationship between the topological space $(\mathcal{S}, \mathcal{T}_o)$ and the informalogical space $(\mathcal{S}, \mathcal{I})$ from which the topological space is derived. An *open set* in the topological space is a generalized open interval in the informalogical space, an *open cover* of $\mathcal{S}$ in the topological space is a generalized open interval cover of $\mathcal{S}$ in the informalogical space, and a *neighborhood* of a point I (actually, a piece of information) in the topological space is a generalized open neighborhood of I in the informalogical space. A *net* in the topological space is an information net in the informalogical space. Once this is clear it is obvious that the various concepts in informalogical spaces that are redefined in this paper using generalized open intervals, generalized open interval covers and generalized open neighborhoods are exactly the same as corresponding concepts in the open interval induced topological spaces. Those concepts include open accumulation information of an information set, open convergence of information nets, open separatedness, open cluster information of an information net, open first countability and open compactness. The following table describes the correspondence between those concepts in informalogical spaces and corresponding concepts in open interval induced topological spaces.

Informalogical space $(\mathcal{S}, \mathcal{I})$	Topological space $(\mathcal{S}, \mathcal{T}_o)$
open accumulation information	accumulation point ([4, p. 41])
open convergence	Moore-Smith convergence ([4, p. 66])
open separated	Hausdorff (or, separated) ([4, p. 67])
open cluster information	cluster point ([4, p. 71])
open first countability	first countability ([4, p. 50])
open compactness	compactness ([4, p. 135])

The purpose of introducing generalized open intervals and related concepts in this paper now becomes clear: It is for building a bridge between informalogical spaces and topological spaces. It is also clear now that all the theorems in [3] that are related to the concepts in the above table can be obtained directly from the corresponding existing results in general topology once the bridge between informalogical spaces and topological spaces is built. This also shows that the concepts in the above table and related results in informalogical spaces have topological nature. With the bridge between informalogical spaces and topological spaces being built, we can apply more existing concepts and results in general topology to the study of informalogy and thus try apply existing concepts and results in general topology to real world applications in information sciences. These fall into our future works.

It is worthy to point out again that an open interval induced topological space can only be established in an open normal informalogical space. This is a limitation of an open interval induced topological space comparing with a closed interval induced topological space which can be established in a general informalogical space. Also, similar to closed interval induced topological spaces, study of only open interval induced topological spaces does not cover all characteristics of the original informalogical spaces.

4. Conclusions and Future Work

In our future work, we will explore using directed graphs to represent an informalogical space, especially when the number of pieces of information in the informalogical space is relatively small.

In our future work, we will also investigate decompositions of information and decompositions of informalogical spaces.

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